

Figure 1: Spin Period and DM Measurement

The spin period and the DM data will be taken from the observation outputs and compared to the ATNF database values for the same pulsar.

Pulsar Parameter Formulas and Derivations

This section utilizes the formulas in the HPA to derive the pulsar parameters. The values that could not be derived were supplemented using the values of the ATNF (2) database.

Period: This value was both measured and provided by the ATNF database.

P-dot (\dot{P}): derivative of the spin period (ATNF)

Frequency (ν) $\nu = \frac{1}{P} \quad (1)$

f-dot $\dot{\nu} = \frac{\dot{P}}{P^2} \quad (2)$

f-dot dot $\ddot{\nu} = \frac{2\dot{P}^2}{P^3} - \frac{\ddot{P}}{P^2} \quad (3)$

The ATNF database does not have \ddot{P} so it is derived from equation 3 as:

P dot dot $\ddot{P} = \frac{2\dot{P}^2}{P} - \ddot{\nu}P^2 \quad (4)$

Spin Down Luminosity:

$$\dot{E} = 4\pi^2 I \dot{P} P^{-3} \cong 3.95 \times 10^{31} \text{ erg s}^{-1} \left(\frac{\dot{P}}{10^{-15}} \right) \left(\frac{P}{s} \right)^{-3} \quad (5)$$

Assumption: $1.4 M_{\text{sun}}$, neutron star with radius 10 km and moment of inertia $I=10^{45} \text{ g cm}^2$.

Breaking Index (n): $n = 2 - \frac{P\ddot{P}}{\dot{P}^2} \quad (6)$

Characteristic Age: $\tau_c = \frac{P}{2\dot{P}} \quad (7)$

Surface Magnetic Field Strength: $B_S = 3.2 \times 10^{19} G \sqrt{P\dot{P}} \quad (8)$

Goldreich-Julian model of the pulsar magnetosphere plasma density at the polar cap:

$$\eta_{GJ} = \frac{\Omega B_S}{2\pi c e} \cong 7 \times 10^{10} \text{ cm}^{-3} \left(\frac{P}{s} \right)^{-1/2} \left(\frac{\dot{P}}{10^{-15}} \right)^{1/2} \quad (9)$$

Polar Cap Radius: $R^P = \sqrt{\frac{2\pi R^3}{cP}} \cong 150 \text{ m} \left(\frac{P}{s} \right)^{-1/2} \quad (10)$

The potential drop between the magnetic pole and the polar cap

$$\Delta\Psi = \frac{B_S \Omega^2 R^3}{2c^2} \cong 2 \times 10^{13} \text{ V} \left(\frac{P}{s} \right)^{-3/2} \left(\frac{\dot{P}}{10^{-15}} \right)^{1/2} \quad (11)$$

Light Cylinder Radius: $R_{LC} = \frac{c}{\Omega} = \frac{cP}{2\pi} \cong 4.77 \times 10^4 km \left(\frac{P}{s}\right)$ (12)

Corresponding dipole magnetic field: $B_{LC} = B_s \left(\frac{\Omega R}{c}\right)^3 \cong 9.2G \left(\frac{P}{s}\right)^{-5/2} \left(\frac{\dot{P}}{10^{-15}}\right)^{1/2}$ (13)

Observations

The DSES team observed three pulsars: B0329+54, B0950+08, and B1133+16. The PRESTO output included a plot and a text file with signal level vs. time for one spin period. The text file was used to plot the raw data and to determine the pulse width $W(50)$.

B0329+54

The presto output of B0329+54 is shown in figure 2. The measured spin period was: 714.50 ms. and the DM was: $27.007 \text{ pc cm}^{-3}$. The ATNF values were: period = 714.5197 ms. and the DM = $26.7641 \text{ pc cm}^{-3}$.

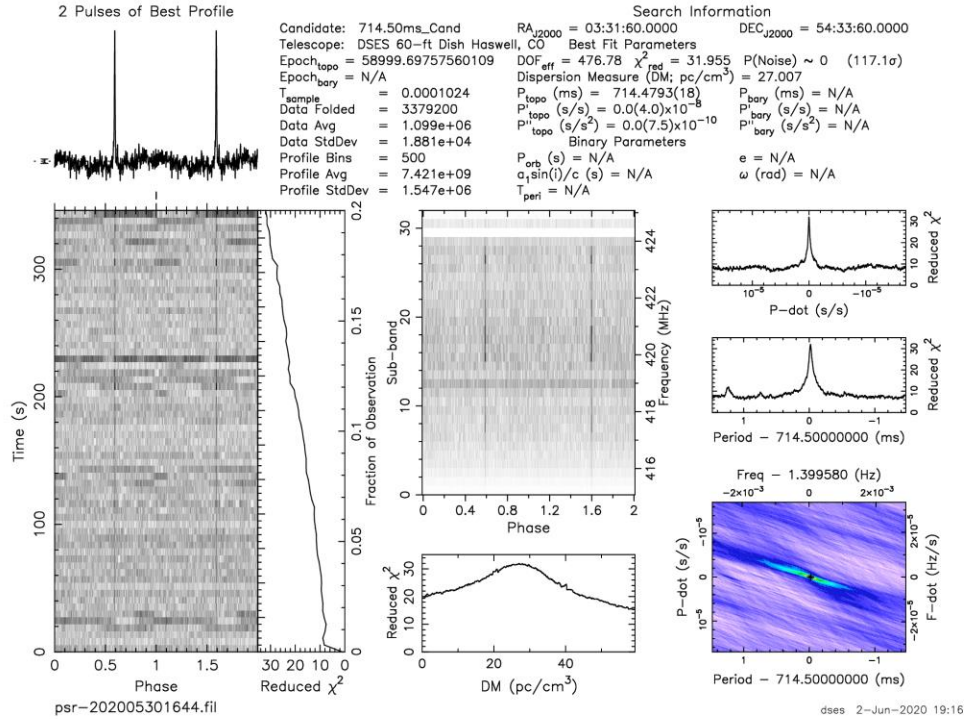


Figure 2: PRESTO Output of B0329+54

The raw text file was plotted for one spin period. (figure 3) This compares with previous observations in the EPN database (3).

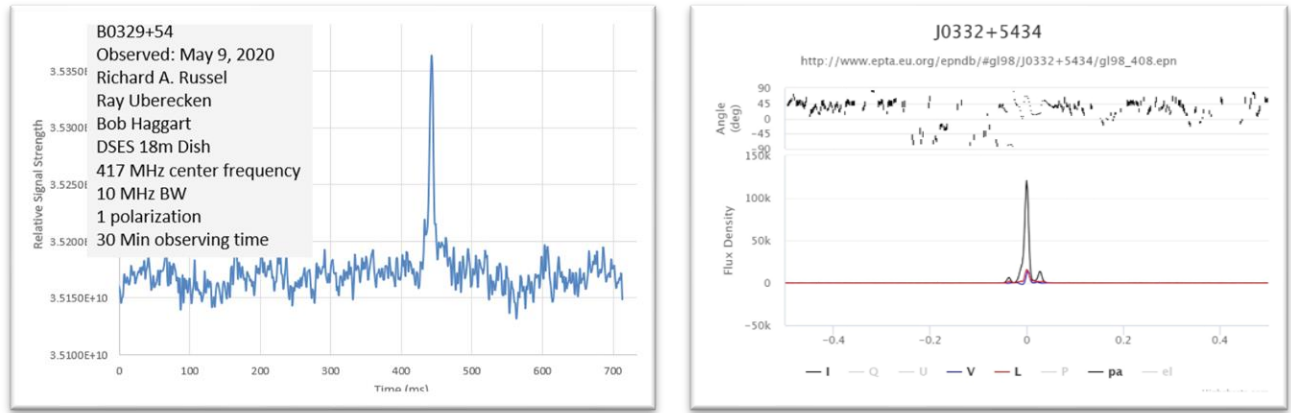


Figure 3: B0329+54 Plot of One Spin Period next to the EPN database observation

The pulse width $W(50)$ can be determined by measuring the width of the pulse at 50% of the pulse height compared to the average noise level. (figure 4)

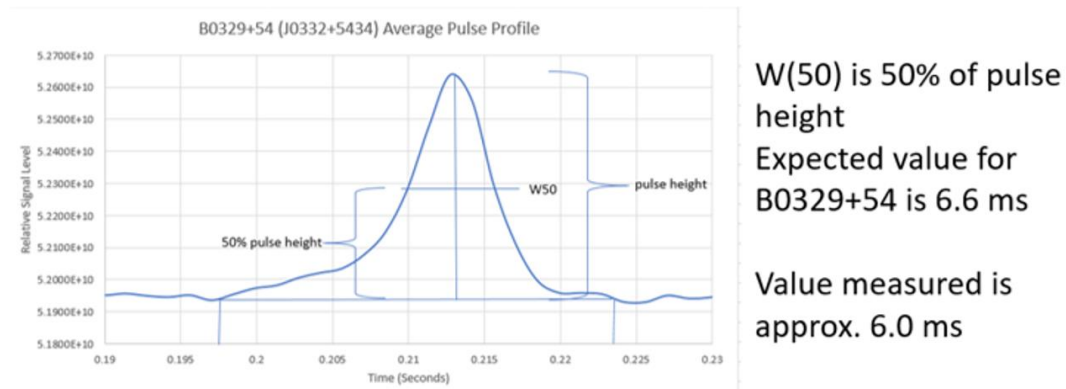


Figure 4: Pulse Width $W(50)$ for B0329+54

Analysis

The observation of B0329+54 corresponded with the pulsar parameters of the ATNF database.

B0950+08

The spin period of B0950+08 was measured to be: 253.06858 ms the DM was measured to be: 2.982 pc cm⁻³. (figure 5). This is compared to the ATNF values of: period =252.58 ms. and DM =2.96393 pc cm⁻³.

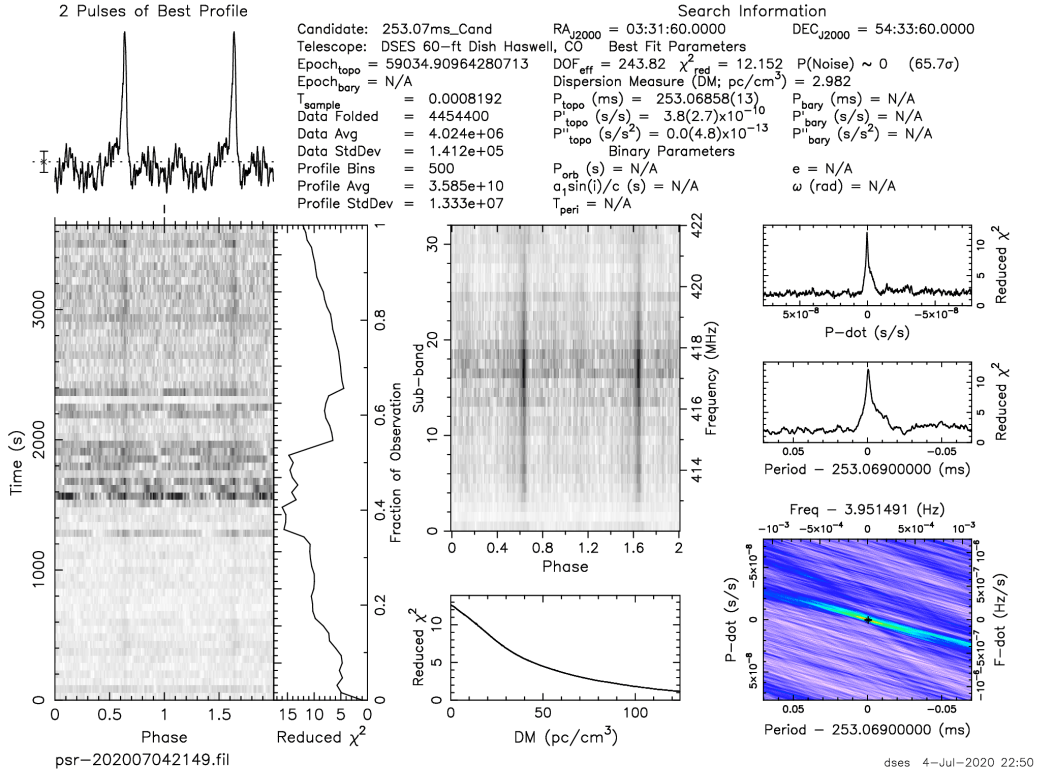


Figure 5: PRESTO Output of B0950+08 Observation

The raw data output for one spin period is plotted in figure 6 along with the EPN (3) database observation.

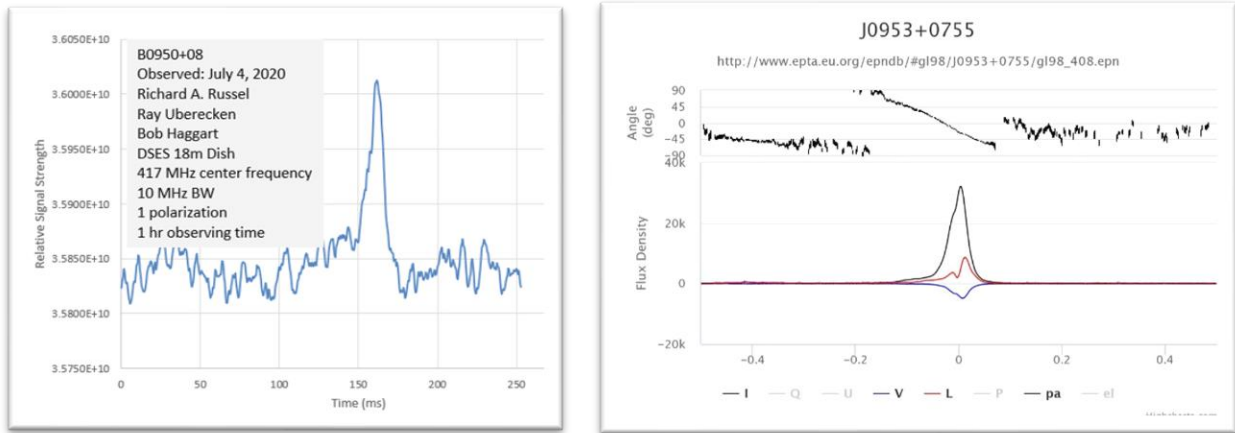


Figure 6: B0950+08 Raw Data Plot of One Spin Period and the EPN database observation

The W(50) value was measured to be 11 ms vs. the ATNF (2) database value of 9.1 ms. (figure 7)

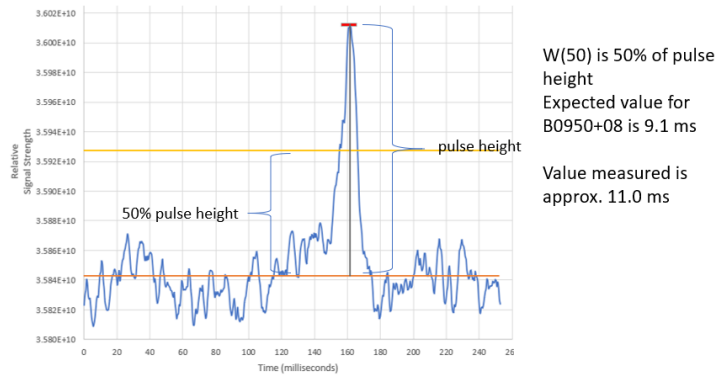


Figure 7: B0950+08 W(50) Plot

Analysis

The observation of B0950+08 corresponded with the pulsar parameters of the ATNF database.

B1133+16

The spin period of B1133+16 was measured to be: 1187.954 ms and the DM was measured to be: 4.86 pc cm⁻³. (figure 8) The ATNF values were: period = 1187.913 ms. and the DM = 4.8407.

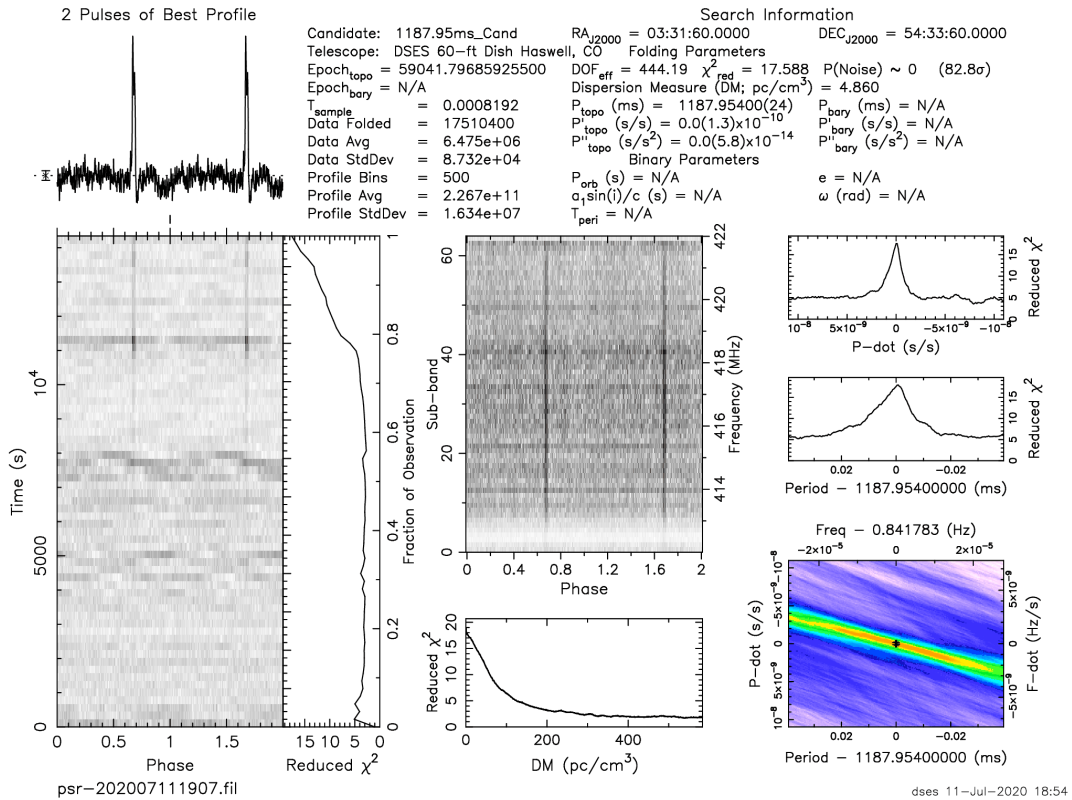


Figure 8: B1133+16 PRESTO Output

The raw data plot for one spin period of B1133+16 is shown in figure 9 along with the EPN (3) database observation.

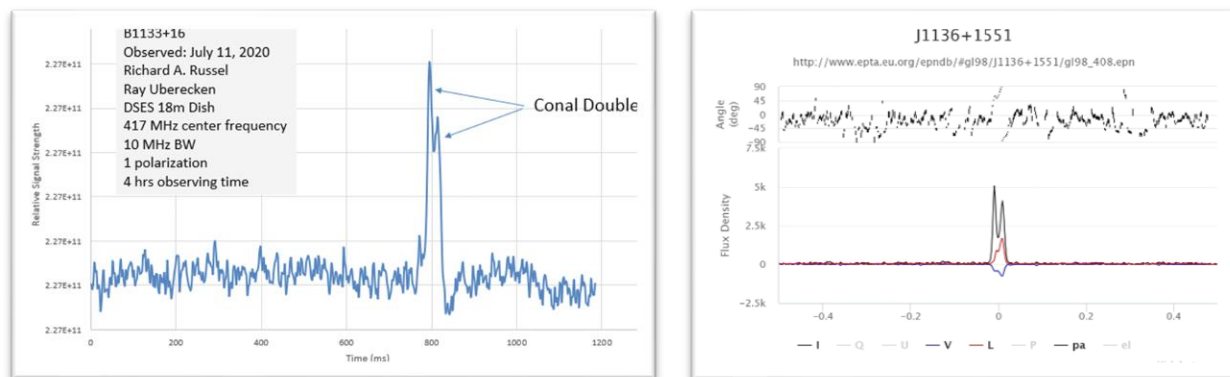


Figure 9: B1133+16 Raw Data Plot of One Spin Period with the EPN (3) database observation

The derivation of W(50) is shown in figure 10. Note that the measured value of 27 ms. is significantly larger than the ATNF database value of 5.9 ms.

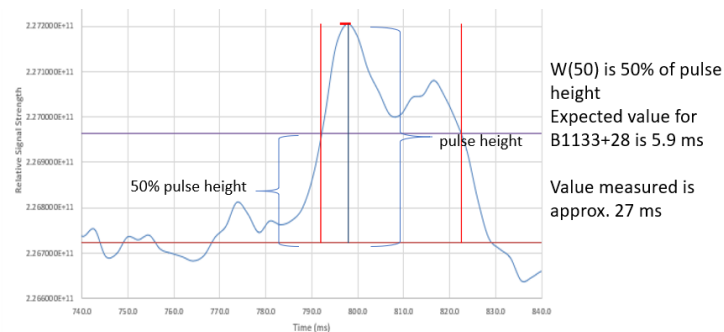


Figure 10: B1133+16 Raw Data W(50) Measurement

The ATNF database W(50) value appears to be for the first pulse. (figure 11) In order for W(50) to be 5.9 ms. for the observation, the pulse height would need to be almost 2x the observed height. This implies that the observation sample rate needs to be faster in order observe the complete signal. This measurement was taken at 10 Mbps so the next observation should be taken at a faster rate to prove this hypothesis.

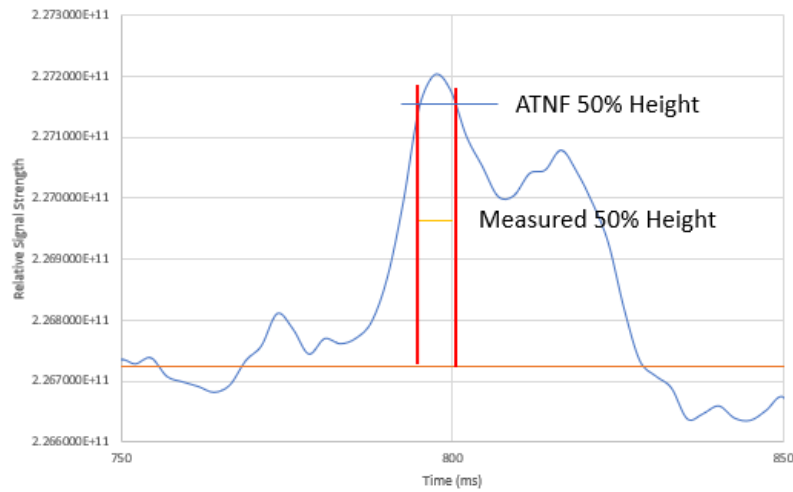


Figure 11: Measured vs ATNF 50% height

The $W(50)$ of the second pulse was measured to be 16 ms. (figure 12).

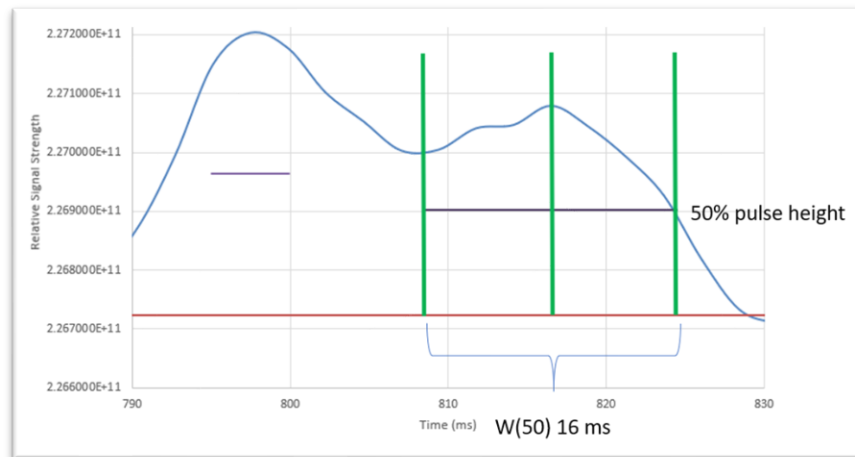


Figure 12: Second Pulse Measured $W(50)$

Analysis

The observed period matched the ATNF database however the pulse width $W(50)$ was significantly larger. The data shown in figure 12, suggests that there are at least 2 pulses very close together. If the first pulse had the ATNF database $W(50)$ value of 5.9 ms, then the observed pulse height was not very accurate.

The proposed second pulse appears to overlap the first pulse. If the assumption of the pulse height above noise is accurate, then the derived pulse width ($W(50)$) is 16.0 ms.

Calculated Parameters

The theoretical geometry of a pulsar is shown in HPA (1). The radius of the light cylinder and the magnetic field (B_{LC}) at the light cylinder radius (R_{LC}) is shown in figure 13.

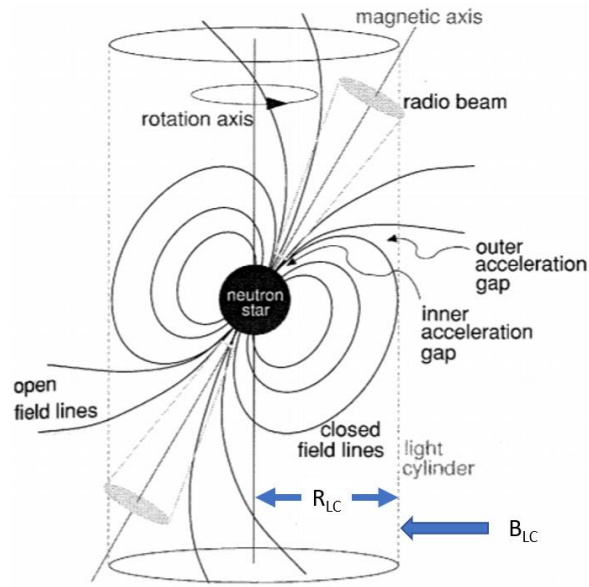


Figure 13: Pulsar Geometry (1)

The science parameters of the pulsars were calculated using the data from the ATNF database and the pulsar formulas of the HPA appendix 2. (figure 14)

Parameter	B0329+54	B0950+08	B1133+16	Units	Equation
P	0.7145197	0.25258	1.187913066	seconds	ATNF
P-dot	2.05E-15	2.30E-16	3.73E-15	seconds/second	ATNF
P-dot-dot	-2.59E-28	4.85E-28	-1.20E-27	seconds/seconds ²	
Dispersion Measure (DM)	26.7641	2.9693	4.8407	pc/cm ³	ATNF
Pulse Width 50% (W(50))	6.6	8.9	5.9	ms	ATNF
f	1.40	3.96	0.84	hz	1
f-dot	4.01E-15	3.60E-15	2.65E-15	s ⁻²	2
f-dot-dot	5.30E-28	-7.60E-27	8.70E-28	s ⁻³	ATNF
Edot (Spin down luminosity)	2.22E+32	5.60E+32	8.80E+31	ergs/second	4
n (braking index)	-4.41E+01	2.34E+03	-1.03E+02		5
age (Myears)	5.53	17.46	5.04	Myears	6
Surface Magnetic Field Strength	1.22E+12	2.44E+11	2.13E+12	Gauss	7
η_{GJ} Goldreich- Julien plasma density	3.75E+03	2.11E+03	3.92E+03	cm ⁻³	8
Rp (polar cap radius)	126.8	75.5	163.5	meters	9
$\Delta\psi$ (potential drop between magnetic pole and polar cap)	1.50E+06	2.38E+06	9.44E+05	V	10
RLC (light cylinder)	3.41E+04	1.21E+04	5.67E+04	km	11
BLC (Light cylinder dipole magnetic field)	9.65E-07	4.33E-06	3.66E-07	Gauss	12

Figure 14: Calculated Parameters

Summary

Formulas for the parameters of a pulsar are shown in the HPA. The amateur radio astronomer has the capability of deriving important pulsar science using these formulas.

This has been an excellent exercise in using radio astronomy observations to derive science. This exercise can be used by professionals and college students alike to characterize pulsars.

Acknowledgements

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References

1. **Lorimer, D. and Kramer, M.** *Handbook of Pulsar Astronomy*. s.l. : Cambridge University Press, 2004.
2. **ATNF Pulsar Database**. (n.d.). Retrieved from <https://www.atnf.csiro.au/research/pulsar/psrcat/>
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Dr. Rich Russel is the vice president for SARA and the current science lead for the Deep Space Exploration Society. He is a retired Northrop Grumman Senior Systems Engineer and served as the Chief Architect for the Satellite Control Network Contract (SCNC). In this capacity he was charged with planning the future architecture of the Air Force Satellite Control Network (AFSCN) and extending the vision to the Integrated Satellite Control Network (ISCN). Dr. Russel has been the lead architect and integrator for the Space-Based Blue Force Tracking project for U.S Space Command, the Center for Y2K Strategic Stability, and CUBEL Peterson. Dr. Russel also has led the SPAWAR Factory team in the deployment of the UHF Follow-On Satellite system. He has a Doctorate in Computer Science, an Engineers Degree in Aeronautics and Astronautics, a Master's in Astronautical Engineering, and a Bachelor's in Electrical Engineering. He is also certified as a Navy Nuclear Engineer and he is a retired Navy nuclear fast attack submariner and Navy Space Systems Engineer.